The effect of imperfect antenna cross-polar performance on the diversity gain of a polarization-diversity receiving system

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Summary

The achievement of diversity gain relies on the realization of two signal branches with low signal correlation. A statistical technique is used to simulate the coupling between the channels and quantifies the effect of cross-polar coupling as a function of the intrinsic correlation of the received signals and the coupling introduced by the antenna. The definition of polarization orthogonality is examined, and both rigorous and approximate methods of measurement are described.

1 Introduction

The use of diversity reception in mobile radio systems relies on the realization of two receive signal paths in which the variation of signal level with time is to some extent uncorrelated. The dependence of the achieved diversity gain on the cor-relation existing between the two paths has been established by a number of investigators (The author's earlier paper, Reference 1, provides an overview and further references). In polarization diversity systems the signal paths are differentiated by the use of two receiving antennas which respond to orthogonally-polarized components of the received signal. It is clear that imperfect cross-polar discrimination at the receiving antennas forms a mechanism that couples the polarization components and increases the correlation between the two branches of the diversity system. This will reduce the achieved diversity gain compared with that realized when using a perfect antenna.

The diversity gain of the system is a function of the correlation between the signals presented to the receivers; their correlation is a function of the transmission path and of the finite cross-polar coupling introduced by the receiving antennas.

2 Modeling the signal branches

The method used by the author was to take two signal branches carrying un-correlated pseudo-random signals with equal mean levels. Two mixing processes were then introduced, one corresponding to the transmission path and the other to the receiving antenna (Figure 1). To model the arriving signals a mixing process is used to increase the correlation between channels to the extent required to model a real transmission environment. The objective is to create a pair of signals with a correlation (Cp) which can be adjusted to take up values typical of a transmission path (not to model the actual processes in a real transmission path, which are entirely different).

For the purpose of simulation the two signal paths contain initial uncorrelated signals a and b. At the end of the first mixing process the signals have a correlation coefficient determined by the mixing fraction r, which is chosen to establish the required path correlation Cp. The second process represents the receiving antenna in which some fraction

s of the orthogonal signal is coupled into the co-polar output. The parameter s is the XPD of the antennas *with respect to the orthogonal reference polarization pair – see Para 5*, and is assumed to have the same value for both channels – a correct assumption for any antenna with symmetrical elements for the two polarizations. (It would be simple to extend the present method to any chosen unsymmetrical pair of coupling factors.)

The simulation was run on a Microsoft Excel spread sheet, using two sets of 1000 pseudo-random numbers to represent the signals on each path, and making use of the correlation function provided by the program. The mixing ratio for the first process was adjusted to achieve successive output correlations of 0.1, 0.2, 0.3… up to 0.7. For each of these values of the signal correlation over the transmission path, the second mixing fraction was given

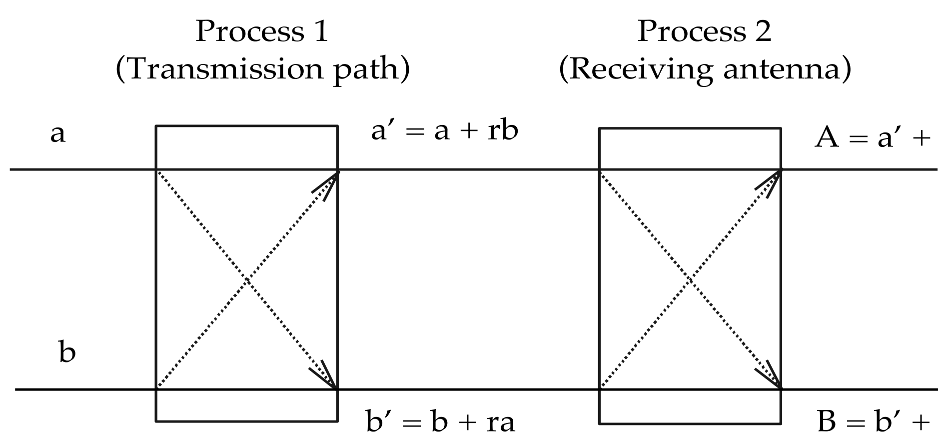

Figure 1: Mixing process 1 is used to create two signal branches with the required correlation. Process 2 represents the imperfect cross-polar discrimination of the receiving antenna.

values representing antenna XPDs of 5dB, 10dB, 15dB … 30dB. Each trial was run with ten sets of 1000 random signal values. The values of path correlation, XPD and resulting output signal correlation were plotted (Figure 2). The slight scatter of the data points results from the statistical nature of the approach used and could be reduced by increasing the number of trials or the number of data points in each trial.

3 Results

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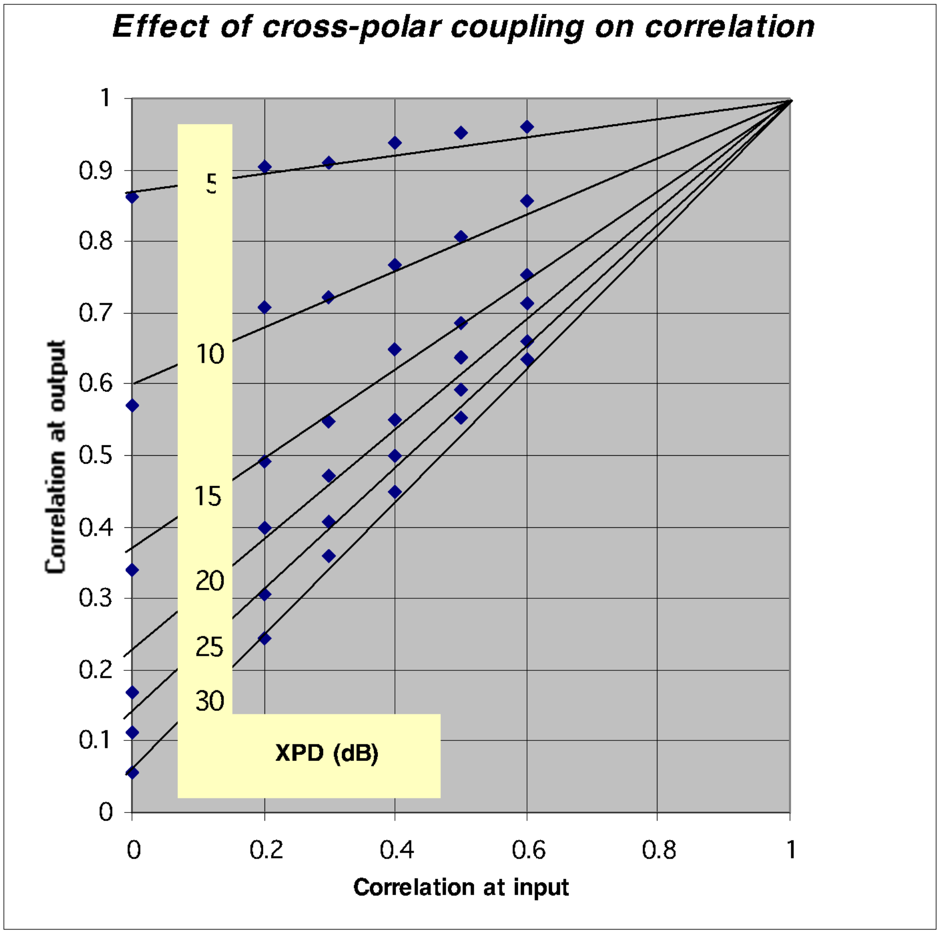


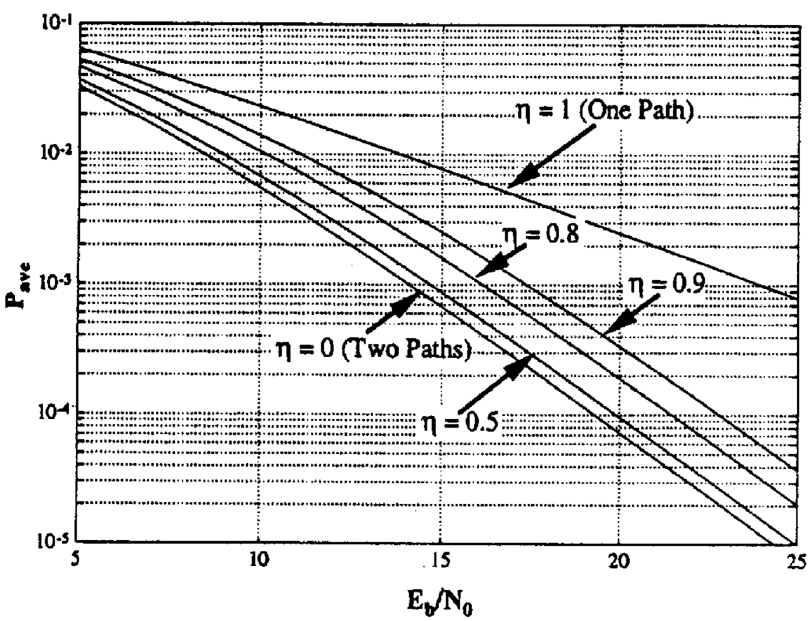
Figure 2: Correlation of incoming signals and resulting output from a dual polar antenna with finite cross-polar discrimination.

The form of the result is not surprising, as three lines could be drawn by examining extreme situations. If the input correlation is 1, then for any antenna XPD the resulting correlation will be 1, establishing a point of convergence for all curves at (1,1). If the antenna has zero XPD, then for any input correlation the output correlation is 1; this establishes the line forming the top of Figure 2 as the result for XPD = 0dB. Finally, if the antenna has perfect XPD, then the resulting correlation of the output signals will be the same as the correlation of the signals in space. Given that all results will be contained within these straight lines, the simple dependence of input and output correlation coefficients is not surprising. When the input signals have low correlation the effect of antenna XPD is large, but as the signal correlation increases the effect of finite antenna XPD diminishes.

The diversity gain of a two-branch system is not a linear function of the correlation of the signals in the two branches. For a signal reliability of 90% the diversity gain of a two-branch system begins to fall rapidly only when the branch signal correlation rises above about 0.7.

To deduce the effect of antenna XPD on diversity gain it is necessary to determine the diversity gain associated with each value of input and output correlation coefficient. This has been done using the curves provided by Ling (1) shown in Figure 3. The change in diversity gain for a signal reliability of 90% produced by the imperfect XPD of the receiving antenna is plotted as a function of the XPD and the incoming signal correlation in Figure 5.

For the sake of completeness, the full relationship between signal branch amplitude inequality and correlation is shown in Figure 4. In a situation in which there is a large signal in one channel and a small one in the other, no effective loss of communication results if some of the large signal is mixed into the low-amplitude channel, even if the result is complete cancellation of the lower signal. When the larger signal fades, the existence of channel coupling has little effect on the low-level signal. For this reason, the consideration of effect of finite XPD on diversity gain contained in this paper concentrates on the situation in which comparable signal amplitudes are present in both diversity branches.



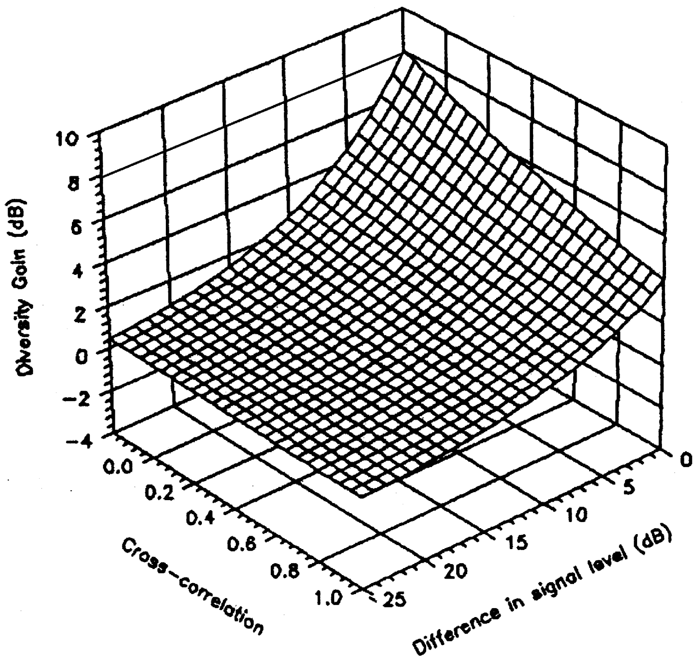
Matched filter bound for QPSK/BPSK signalling – two correlated paths

η = Coeff of correlation

= Cov(x,y)/sqrt [Var(x).Var(y)]

MFB is a bound on the optimal performance of a communications channel that assumes no ISI and a known channel

Figure 3: Relationship between diversity gain and signal path correlation (Ling, 1995)



Diversity gain at 90% signal reliability as a function of cross- correlation and mean branch signal level difference. (Maximal ratio combining)

Figure 4: Relationship between diversity gain and the relative levels and correlation of the signals in a 2-branch diversity system with maximal-ratio combining (Turkmani et al).

4 The effect of XPD on diversity gain

The imperfect cross-polar discrimination of a practical dual-polar receiving antenna will cause a small reduction in the diversity gain which could potentially be obtained by exploiting the partially-uncorrelated nature of the signal components received with orthogonal polarizations.

A typical loss of diversity gain of 0.5dB is produced by an antenna XPD of around 17dB, and a loss of 1dB by an antenna XPD of 12dB. On boresight a typical dual-polar antenna will provide an XPD of 20dB, while at the sector edge (60° from boresight) the XPD will have fallen to around 12dB. The consequent 0.5dB fall in diversity gain between the center and edge of the sector is comparable with that in a typical space-diversity system, where the lateral spacing of the receiving antennas decreases as the received signal moves off the axis of the antennas (see Reference 3).

The extent of lost diversity gain is only weakly dependent on the correlation of the incoming signal – as the signal correlation increases from 0 to 0.6, the effect of finite XPD typically reduces the available diversity gain by about 0.3dB.

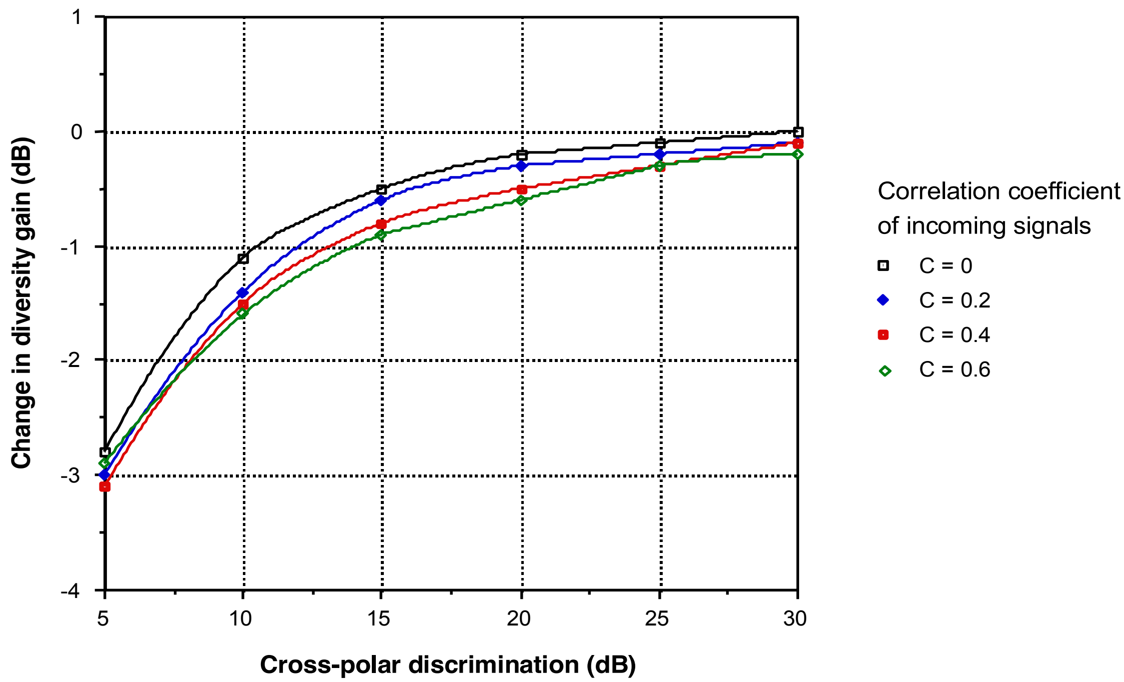


Figure 5: Reduction in diversity gain as a function of antenna XPD and the correlation of the incoming orthogonal signal components.

5 Definitions and measurement methods

In the analysis above the term *cross-polar discrimination* has been used without careful definition of its method of measurement. The polarization performance of a base-station receiving antenna varies as a function of the angle of arrival of the received signal; a real antenna radiates (or optimally receives) an elliptically-polarized wave with a particular orientation and eccentricity. The polarization ellipse of the signal changes as a function of frequency, and of direction in azimuth and elevation. For most types of slant-polar base-station antenna the polarization angle is close to 45° on boresight, but as the observer moves from the boresight the antenna polarization moves towards the vertical – the polarizations received by the two ports converge.

There are several different methods of measuring the polarization performance of a base station antenna. Complete characterization can be carried out only by measuring the complex components of the field radiated/received by the antenna (E_V and E_H) and computing the polarization ellipse for each spacial direction and frequency.

Experiments carried out by others to establish the usefulness of polarization diversity have assumed that the receiving antenna responds to orthogonal linearly-polarized signal components with polarization angles of +45° and -45°. For this reason – and because of the simplification of the required measurements – the author's team carries out all measurements using plane-polar illumination with polarization angles of +45° and -45°. Measurements of radiation patterns in which the antenna under test is illuminated in turn with these two orthogonal polarizations provide a description of polarization behavior which is sufficient to allow the effect of polarization performance on diversity gain to be defined. It is simple to carry out and requires no complex assumptions or calculations. Measurement methods are described below in more detail.

The use of the term cross-polar discrimination (XPD) in this paper differs slightly from its general use. The reference polarizations are taken to be a fixed orthogonal pair of linear polarizations (not the exact co-and cross-polar planes for the two parts of the antenna under test, as the two co-polar planes may not be mutually at right angles). The result is a simple and self-consistent set of definitions and parameters which, as is shown in this paper, relate easily to the propagation studies carried out by others.

Some antenna specifications include *diversity gain* as an antenna parameter. As diversity gain is a function of the signal transmission path (and a defined level of signal reliability) as well as antenna performance and combining method, this is not appropriate. Separate definitions of orthogonality are not needed if the polarization behavior is measured using orthogonal fields as described above. Measurements of radiation patterns using linear reference polarizations of +45° and -45° generally provide adequate data to characterize the cross-polar performance of base station antennas.

We now review the concept of polarization orthogonality and examine two methods of measurement, one rigorous and the other simpler but more approximate.

6 The phenomenon of polarization

Most radio engineers are familiar with the concept of polarization as describing the plane of the electric field of an electromagnetic wave. We generally imagine waves as essentially linearly polarized, with a few special applications in which the wave is circularly polarized.

By contrast, radio astronomers (and many HF radio engineers) deal with received signals with polarizations which are often entirely arbitrary and from which no single real antenna can abstract all the energy incident on the effective aperture of the antenna.

Any electromagnetic wave can be envisioned as having polarization components as shown in Figure 6.

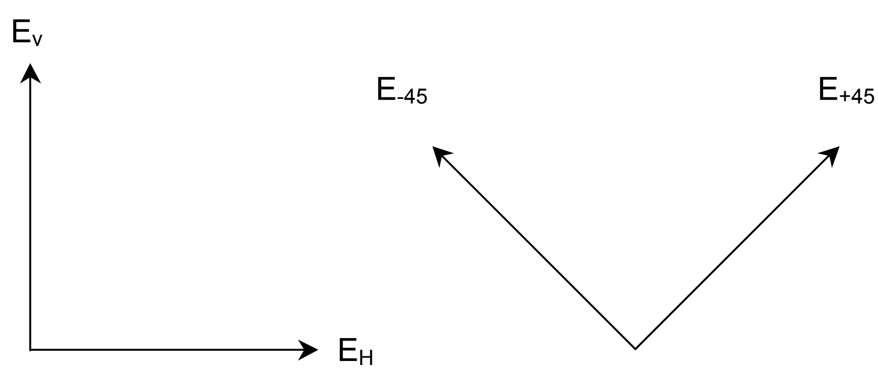


Figure 6: In the mobile radio environment the two field components are relatively independent of one another in amplitude and phase. In general it is not possible to sum the vectors into a single direction containing the whole signal power because the vectors are not mutually in phase. The magnitude and phase relationship of the vectors changes rapidly with time, so the sum vector traces out an ellipse whose eccentricity and orientation depends on the relative magnitudes and phases of the components.

To simplify the notation we imagine a wave traveling parallel with the surface of the ground and characterize the electric field by two components E_v and E_h, mutually at right angles. These have some phase difference θ, which we can determine by measurement. Because of this phase difference we cannot simply sum the two field components by drawing the usual simple vector sum.

6.1 Special cases

There are two special cases of polarization state with which everyone is familiar. When describing polarization the standard convention is to look at the signal in the direction of propagation – as if the observer is standing behind the transmitting antenna.

6.1.1 Linear polarization

When $\theta = 0°$ the two components are in phase and we can sum the components in the usual way. This is the special case of linear polarization. The plane of polarization will be determined by the relative magnitude of the two components E_V and E_H and we can define a polarization angle as $\phi = \mathrm{Tan}^{-1} (E_V/E_H)$.

If $E_V = 0$, then the signal polarization is described only by E_H, so the signal is horizontally polarized. Similarly when $E_H = 0$ the signal is vertically polarized. If $E_V = E_H$ (and at the same time $\theta = 0°$) the signal is polarized at 45° and if $E_V = - E_H$ (ie $\theta = 180°$) the polarization is -45°.

6.1.2 Circular polarization

If $|E_V| = |E_H|$ and the two vectors are in phase quadrature (±90°), the resulting signal is circularly polarized, the sense of rotation depending on the + or - sign.

6.2 The general case

Apart from these two special cases, all other signals are elliptically polarized, having some relation E_V/E_H and phase θ between them. As the wave propagates, the electric vector traces out an ellipse around the axis of propagation.

6.3 The polarization ellipse

As an alternative to defining a wave by E_V/E_H and θ, an entirely equivalent description is obtained by specifying the polarization ellipse by its ellipticity – the ratio of major to minor axes – and the physical orientation of the major axis in space (see Figure 7). The equations relating the two descriptions are set out in the textbooks and the conversion between them can be made graphically using a Carter Chart (Ref 4).

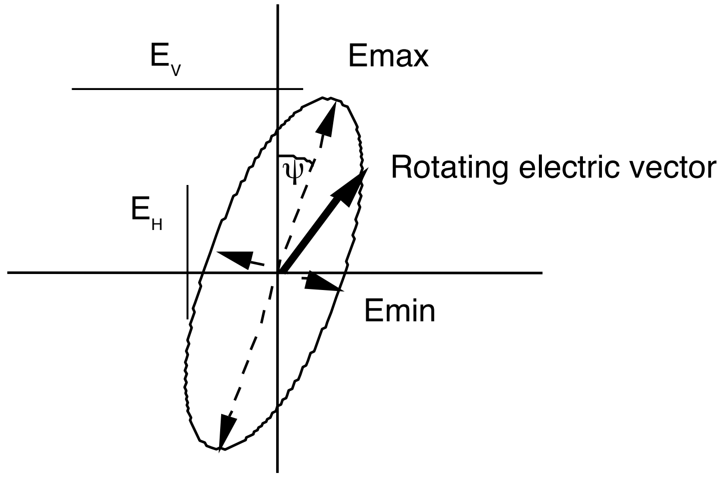


Figure 7: The general case of elliptical polarization. The electric vector of the propagating field traces out an ellipse, having maximum value Emax, and a minimum value Emin one quarter period later. The ellipse is inclined at an angle ψ to the vertical. ψ is known as the polarization angle. When the sense of rotation of the electric vector is clockwise, viewed in the direction of propagation, the polarization is referred to as right-hand. It may be noted that the components E_V and E_H reach their maximum values at two points which are not on the same diameter of the ellipse – in this example they are about 1/3 of a period apart.

The two representations of the polarization of a wave described above are simply alternative descriptions of the same phenomenon. Any polarization can be described in terms of two superposed linear (H/V) components or two superposed circular (RH/LH) components, or by explicit reference to the ellipticity (axial ratio) and polarization angle.

7 Receiving a signal with arbitrary polarization: orthogonality

To receive a linearly polarized signal we know that all we have to do is to orient the receiving antenna so its polarization is aligned with that of the incoming wave. We will then receive all the energy in the wave (as defined by the Poynting vector) and nothing is wasted.

A circularly polarized (CP) signal is used in two ways. We know we can receive all the energy using a CP antenna with the correct handedness; alternatively we can use a randomly-oriented linear antenna and receive half the incident energy.

We know that if we place a vertically-polarized receiving antenna in a horizontally-polarized incident field we will receive nothing. Similarly a right-handed CP antenna placed in a left-hand CP field receives nothing (assuming in each case that the fields and antennas have pure polarization). The combinations of linear polarizations which are mutually at right angles in space, and left/right CP, are orthogonal, a mathematical term implying independence. Clearly, in the linearly-polarized case this mathematical orthogonality is related to the spacial orthogonality of the two polarizations; in the CP example a spacial meaning of orthogonality is less obvious.

How are we to define the orthogonality in the general case of arbitrary elliptically-polarized waves and antennas? We can make a polarization-matched antenna using two crossed linearly-polarized antennas such as dipoles, feeding the two elements through an adjustable power divider and phase shifter. With the right settings of the power divider and phase shifter we can exactly match the polarization of any incoming wave and extract all the power carried by it; by some other setting we can receive no power at all (Refs. 5 and 6).

When a signal is transmitted by one antenna and received by another, the ratio of the received signal to that which would be received by an antenna which is exactly polarization-matched is given by the following equation in which all field components are complex quantities:

$$\frac{P}{P_{max}} = \frac{\left|E1_{+45}.E2_{+45} + E1_{-45}.E2_{-45}\right|}{\sqrt{E1_{+45}.E1^{*}_{+45} + E1_{-45}.E1^{*}_{-45}}\sqrt{E2_{+45}.E2^{*}_{+45} + E2_{-45}.E2^{*}_{-45}}} \qquad (1)$$

where $E1_{+45}$ is the +45° field component from Port 1 and $E2_{+45}$ the +45° field component from Port 2. $E1_{-45}$ is the -45° field component from Port 1 and $E2_{-45}$ the -45° field component from Port 2. E* is the complex conjugate of E.

This equation is essentially unchanged for any pair of orthogonal signal components, whether H/V linear, ±45° linear, or right/left CP. (For H/V, simply substitute E_H for E_{+45} and E_V for E_{-45}; for CP substitute E_R for E_{+45} and E_L for E_{-45}.

In the special case in which the polarization of the receiving antenna is such that zero power is received, its polarization is said to be *orthogonal* to that of the incoming wave.

Care must be taken when referring to the polarizations radiated and received from any specific antenna. The sense of the polarization angle is reversed when the radiated wave is viewed from the position of the receiving antennas, so antennas transmit and receive signals with this sign change. Only the special cases of vertical, horizontal and circular polarizations does this sign change make no difference, so an antenna transmits and receives with the same polarization.

8 Application to mobile radio base station antennas

In the mobile radio base station environment we know that the received signal was radiated by some (generally randomly-oriented) linearly-polarized antenna, and has then been scattered and reflected in transmission. The received signal has a time-varying polarization. Experiments (Ref. 7) have shown that if we arrange the receiving antenna to respond to linear components of the field resolved in the ±45° planes, the two resulting signals will have sensibly equal mean amplitudes and a correlation which is low enough to provide useful diversity gain.

We require a measurement to show us the ‘goodness’ of the polarization response of the receiving antenna. The antenna port labelled +45° must respond to the +45° field component and not (or at least substantially less) to the -45° field component. This independence must be maintained to a useful extent over the 120° azimuth sector covered by the antenna.

The simplest test is to measure the response of the +45° antenna to the +45° component of the incoming wave and to compare this with its response to an incoming wave with -45° polarization. This measurement is effectively a cross-polar discrimination (XPD) measurement in which the ±45° axes are fixed as the nominal polarization axes. It measures (correctly) the relative response of the antennas to two specific orthogonal field components, but especially off-axis the ±45° test polarizations may not correspond to the matched polarizations of the antenna, so the result is less than the true orthogonality.

A more complex and rigorous measurement establishes the mathematical orthogonality of the responses of the two halves of the antenna using Equation 1 above. To obtain the data needed to calculate orthogonality at various bearings from the antenna, the relative phases and amplitudes of E_{+45} and E_{-45} must be measured for both ports of the antenna at various directions in space over the sector of coverage. This is most conveniently achieved using a near-field measurement system and the results computed using a suitable spread-sheet. This method allows for the ellipticity of the polarization response of the antenna. In a typical dual-polar base station antenna, the elements for the two polarizations are generally mirror-symmetrical and have opposite handedness in their polarization responses. For this reason the numerical value of rigorously-measured orthogonality is usually found to be larger than the relative response measured by the simpler ±45° test.

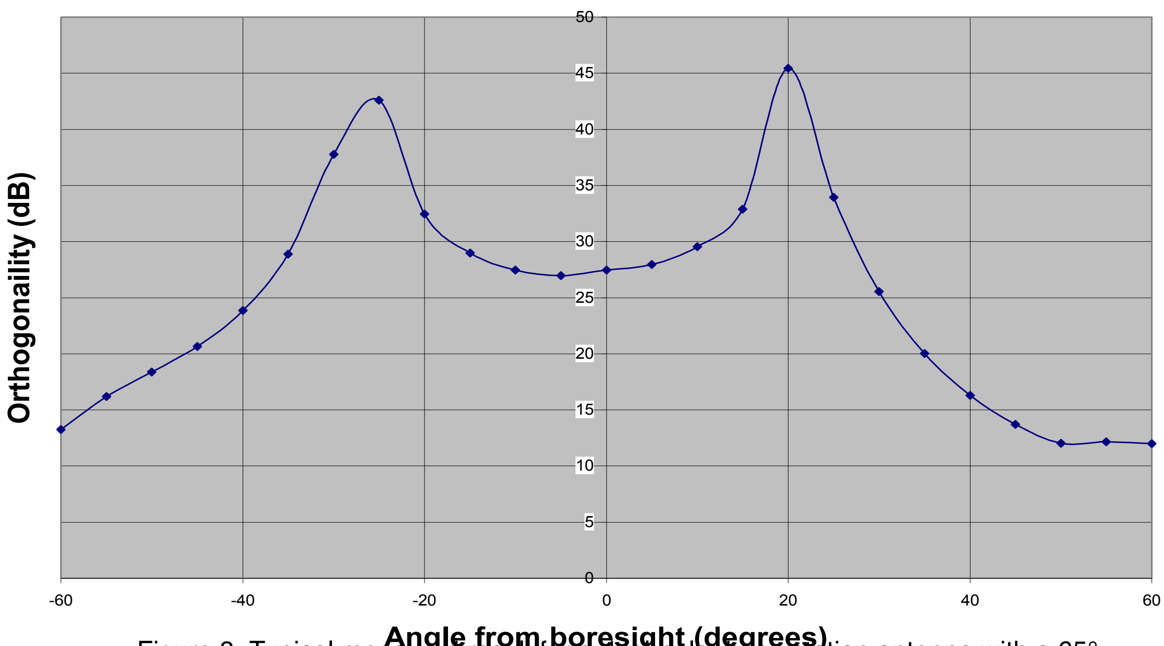


Figure 8: Typical measured result for a dual-polar base station antenna with a 65° azmuth 3-dB beamwidth

Figure 8 shows a typical orthogonality measurement for a 65° sector antenna. The orthogonality exceeds 20dB over much of the sector, falling to around 12dB at +/- 60°. The two received polarizations fold slowly towards vertical polarization as the angle from boresight reaches 90° and the orthogonality tends towards zero.

9 Less adequate definitions

Other definitions of polarization orthogonality have been encountered by the author, with correspondingly different results and methods of measurement.

Some groups measure the angle between the major axes of the polarization ellipses characteristic of the two antennas. This technique is related to the simpler method described above but is less direct and needs further measurements to correctly define the resulting cross-polar coupling effects. Polarization orthogonality is a power ratio and not an angle.

Definitions relating to the dot-product of simple time-invariant vectors are not correct because they do not account in the proper manner for the phase difference between the polarization components. The only complete expression is that shown in Equation 1, and equivalents derived from it in which the reference axes have been changed.

The discrimination of the receiving antenna to most other pairs of orthogonal polarizations will be less than that for the ±45° test polarizations described above, for example, discrimination to orthogonal H and V signals is zero. (If the intention had been for the antenna to respond to and discriminate between H and V polarizations, its design would have been different.) The incoming real-world signal can be resolved into any chosen pair of orthogonal components, and it is the relative response of the receiving antenna to the best-resolved pair which is significant to system operation.

10 Conclusion

The effects of imperfect polarization response of a dual-polar base station antenna typically cause a reduction in the diversity gain that could be achieved by a perfect antenna. In a typical environment the available diversity gain is reduced by 1dB only as the orthogonality of the antennas falls below 10dB.

A full definition and measurement of polarization orthogonality requires measurements of four complex field components radiated/received by a typical ±45° base station antenna. Because of the relatively small effect of orthogonality on diversity gain, a simpler measurement of the relative response of a base station antenna to ±45° linearly-polarized signals is usually adequate.

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